TRINITY COLLEGE

MATHEMATICS
METHODS
UNIT 3,4

Section One:
Calculator-free

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	97	65
				Total	100

Instructions to candidates

1.   The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3.   You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4.   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5.   It is recommended that you do not use pencil, except in diagrams.

6.   Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7.   The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free   **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1   **(5 marks)**

A small body is initially at the origin. It is moving along the x-axis with velocity at time t seconds given by

$$v(t) = (t-2)^3 \quad cm/s$$

(a)   Determine $x(t)$, a function for the displacement of the body at time t.   (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

The small body is stationary when $t = T$.

(b)   Determine the displacement of the body at $T + 3$ seconds.   (2 marks)

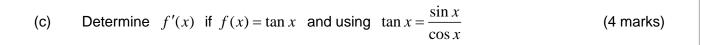**Question 2**   **(10 marks)**

(a)   Determine $\dfrac{d}{dx}\sin(\cos x)$ (2 marks)

(b)   Evaluate $f'\left(\dfrac{\pi}{4}\right)$ when $f(x) = \dfrac{x + \cos x}{\sin 2x}$.   (4 marks)

(c)   Determine $f'(x)$ if $f(x) = \tan x$ and using $\tan x = \dfrac{\sin x}{\cos x}$ (4 marks)

See next page

Question 3 **(8 marks)**

(a) State three key characteristics of a chance experiment that make it suitable for modelling
by a binomial random variable. (3 marks)

Research has shown that 30% of dogs aged over 13 years have some form of heart disease. A
random sample of 40 dogs is selected from a large population of dogs of this age. Let X be the
number of dogs in the sample with some form of heart disease.

(b) Explain why randomly selecting one dog and recording whether it has some form of heart
disease is a Bernoulli trial. (1 mark)

(c) Write a numerical expression for the probability that 11 dogs in the sample have some
form of heart disease. (2 marks)

(d) State the mean and variance of X. (2 marks)

Question 4   **(6 marks)**

Determine the area of the finite region bounded by $y = \sqrt{2x}$ and $y = \dfrac{x}{2}$.

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Question 5   **(7 marks)**

A four-sided die has faces marked with the numbers $1, 2, 2$ and 3. All faces have an equal chance of landing face down after the die is rolled. A game, that costs \$2 to play, involves throwing the die twice and adding the two numbers that land face down. If the total score is 5, the player wins \$7, and otherwise they win nothing.

Let X be the total score obtained in one play of the game.

(a)   Construct a probability distribution table for X.   (3 marks)

(b)   Determine $E(X)$.   (1 mark)

Let Y be the net monetary loss, in dollars, of a player in **two** plays of the game.

(c)   Determine $E(Y)$.   (3 marks)

Question 6   **(5 marks)**

(a)   Determine $\dfrac{d}{dx} 6xe^{\frac{x}{2}}$ (2 marks)

(b)   Hence, or otherwise, determine $\int 6xe^{\frac{x}{2}} dx$ (3 marks)

See next page

Question 7   **(7 marks)**

The following table shows the probability distribution for the random variable T.

t	0	1
$P(T = t)$	$\dfrac{k}{4} + \dfrac{1}{10}$	$\dfrac{13}{10} - \dfrac{1}{5k}$

(a)   Determine the value of the positive constant k and hence state $P(T = 1)$.   (4 marks)

The random variable $W = 5T - 1$.

(b)   Determine $E(W)$ and $\mathrm{Var}(W)$.   (3 marks)

Question 8 **(4 marks)**

The function f is defined by $f(x) = \dfrac{3}{x^2 + 27}$, so that $f''(x) = \dfrac{18(x^2 - 9)}{(x^2 + 27)^3}$.

Describe the concavity of the graph of $y = f(x)$.

End of questions

Supplementary page

Question number: _____